The effect of confining walls on the stability of the steady wake behind a circular cylinder

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The results of an experimental investigation are presented to show that the stability of the steady laminar wake behind a circular cylinder is strongly influenced by the proximity of the walls of the confining experimental equipment.

Introduction

Although it has been known for some time that the measured lift and drag of solid bodies can be influenced by the proximity of the walls of the confining experimental equipment, it has also been believed that such wall effects are of a secondary nature. Thus, for example, when a cylinder is placed transverse to the flow, the presence of the walls causes an increase in the drag of about 18 % if d/h, the cylinder diameter to channel width ratio, is $1/5$, and if the static pressure gradient in the direction of flow is negligible (Pankhurst & Holder 1952). It will presently be shown, however, that in at least one case, the onset of instability of the steady laminar wake behind a circular cylinder, the influence of the confining walls can indeed be dominating, and that at values of d/h as low as $1/5$ the experimental results can be affected in a much more pronounced manner than one would have expected up to now.

The stability phenomenon which was studied in this work concerns the behaviour of the two steady standing vortices that are formed behind a circular cylinder in uniform flow above a Reynolds number, $Re = Ud/\nu$, of approximately 5 (Thom 1953), where U is the speed of the fluid far from the cylinder and ν its kinematic viscosity. These steady vortices increase in size with increasing Reynolds number up to a critical Reynolds number, Re_c, at which fluctuations set in. The fluctuations then dominate the character of the wake and, upon a further increase in Reynolds number, the familiar Kármán vortex street is eventually established. It is the purpose this paper to present the salient features of an experimental investigation of the effect of the confining walls on the stability of the steady wake, and to point out some rather unexpected results which were observed.

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Experimental equipment and methods

The experimental equipment was as described by Shah, Petersen & Acrivos (1962). Essentially, it consisted of a closed tunnel in which oil was recirculated by means of a variable-speed pump. The tunnel had a Lucite test-section, shown in figure 1, with four pairs of port-holes into which the cylinders, ranging in diameter from $\frac{1}{2}$ in. to 2 in., could be placed. Throughout the entire range of the experiments the flow in the tunnel was fully laminar. Typical velocity profiles along the vertical centre-plane of the test-section and just upstream of the first and fourth portholes, respectively, are shown in figure 2. (The measured velocity profiles along the horizontal centre-plane are similar.) These profiles, which were obtained in the absence of the cylinders by means of a photographic bubble tracer technique (described in detail by Shair 1963), clearly demonstrate the existence of a rather sizeable acceleration effect along the central portion of the test-section which is evidently due to the boundary-layer build-up along the wall.

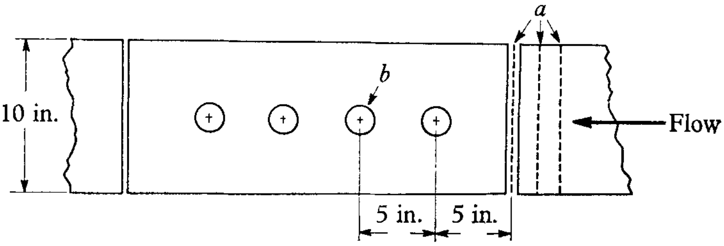

FIGURE 1. Test-section, 8 in. wide; *a*, wire-gauze screens; *b*, 2 in. diameter portholes.

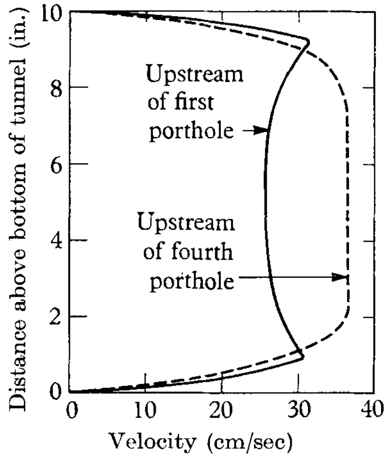

FIGURE 2. Typical velocity distribution in the test-section, along the vertical centre plane.

One of the major difficulties which had to be overcome before the experimental results could be interpreted quantitatively was the proper selection of the characteristic velocity, U, for the cylinder Reynolds number. As explained above, a constant free-stream velocity, the natural choice for U, did not exist because of the acceleration effects within the tunnel. It was found, however, that this difficulty could be resolved by choosing U for a given experiment to be the velocity which would exist at the same location as that of the centre of the cylinder, under flow conditions identical with those of the experiment but in the absence of the cylinder. Experimentally measured front stagnation pressures based on the characteristic velocity, U, as defined above, were shown to agree with theoretical predictions (Grove 1963).

The point of onset of instability was established by placing the cylinder in one of the portholes and slowly increasing the speed of the pump until the motion of very small air bubbles within the standing vortices behind the cylinder became slightly irregular. Each measurement was repeated three to five times with the oil at different temperatures (hence different viscosities) and the critical Reynolds number, Re_c, obtained by this procedure was found to be reproducible to within $\pm 5\%$.

Results and discussion

The results of the experiments described above are presented in figure 3, which illustrates the effect of the confining walls on the stability of the steady wake by showing the variation of the critical Reynolds number with the d/h ratio. For comparison, the results of earlier investigators are also given. Kovasznay (1949) and Roshko (1954) carried out their measurements with a wire of small diameter in an air stream so that their values can be considered as corresponding to the limiting case $d/h \to 0$. Thom (1933) and Homann (1936) experimented with open oil channels where three-dimensional effects are undoubtedly more pronounced than in a closed tunnel. Dupin & Teissiè-Solier (1928) reported no variation in the critical Reynolds number for the range of cylinder sizes which they used, and the two appropriate points on figure 3 correspond, respectively, to the smallest and the largest d/h ratio in their experiments.

These results definitely demonstrate that the stability of the steady wake is greatly enhanced by an increase in d/h. Qualitatively, this would perhaps have been expected since it is known that the wake instability is caused primarily by disturbances which are generated in a direction perpendicular to both that of the undisturbed flow and the axis of the cylinder. Thus it would appear reasonable to expect that the propagation of such disturbances should be inhibited by the presence of the walls which confine the streamlines near the cylinder.

In addition to this genuine wall effect, however, there exists the possibility that at least part of this stabilization may be due to the acceleration of the fluid along the centre-line of the tunnel. In the present work, for example, it was found that the measured form drag on the cylinder was about 50% higher when $d/h = 0{\cdot}2$ than when $d/h = 0{\cdot}05$, and it was concluded, on the basis of the correlations in Pankhurst & Holder (1952), that the contributions of the wall effect and the static pressure gradient to this increase were of approximately equal importance. One might be tempted therefore to attribute the observed increase of Re_c with d/h to the combined effect of both these factors.

Although this point could not be answered with certainty during the course of this investigation, some experimental evidence was obtained which indicated that the presence of a static pressure gradient did not affect the stability of the wake nearly as much as the genuine wall effect. This may be deduced from figure 4, which shows how the critical Reynolds number is influenced by the position of a given cylinder within the test section. The change in Re_c evidenced in figure 4 is caused at least partly by the variation of the static pressure *gradient* along the

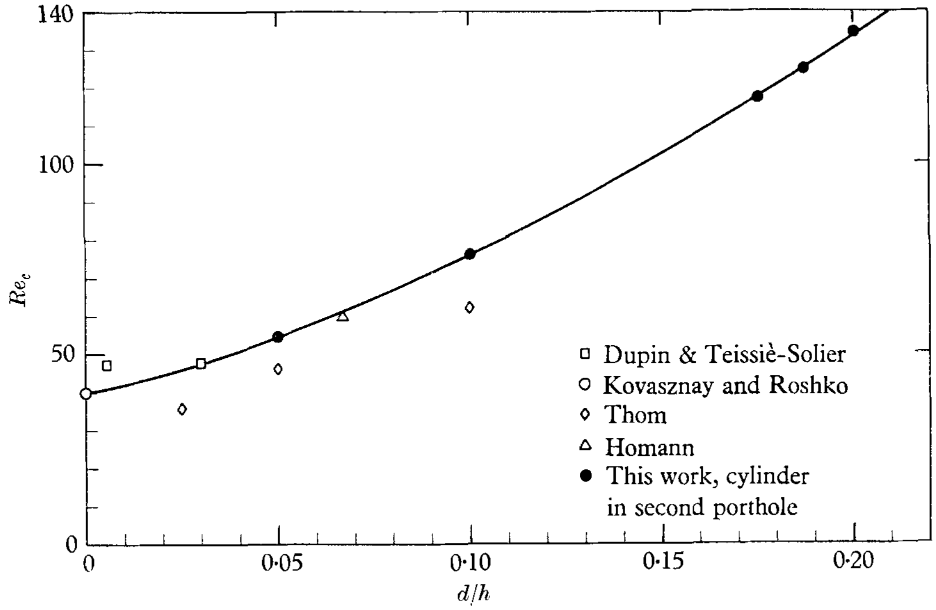

FIGURE 3. Effect of the confining walls on the stability of the wake.

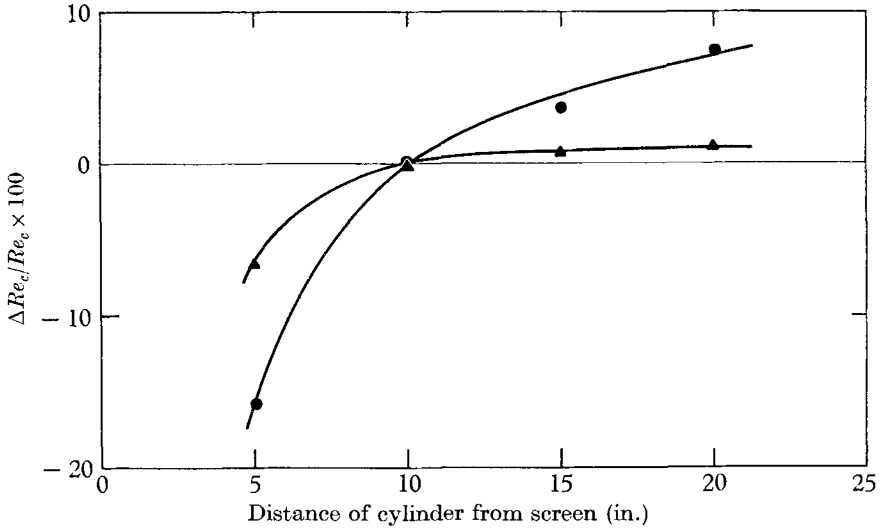

FIGURE 4. Effect of the position of the cylinder within the test-section on the stability of the wake. Re_c = critical Re observed with cylinder in the second porthole. ΔRe_c = critical Re observed with cylinder in a given position minus critical Re observed with cylinder in the second porthole. ▲, $d/h = 0·05$; ●, $d/h = 0·20$.

test section but, as is easily seen, the relative magnitude of this charge is substantially smaller than the dependence of Re_c on d/h which can be observed in figure 3. (Owing to the presence of the screens, the static pressure gradient was smallest near the entrance to the test section, Grove 1963.) This inference concerning the effect of the static pressure gradient on the stability of the wake is, however, only tentative and should, if possible, be substantiated by means of further experimental work.

The most important conclusion to be drawn from the results of this paper is that, whereas the influence of the confining walls on phenomena associated with the flow around a solid object is frequently of a secondary nature, the presence of these walls can drastically affect the stability of the steady wake behind a circular cylinder. When $d/h = 0{\cdot}2$, for example, the critical Reynolds number is increased by more than 300 %, which is much larger than the corresponding change in other parameters of the flow, such as the form drag.

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